HIGH-FREQUENCY PLASMA OSCILLATIONS IN JUNCTION DIODES

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Longitudinal high-frequency plasma oscillations in junction diodes are examined by the kinetic equation method with allowance for the asymmetry of the boundary conditions at the boundaries between the plasma and the electrode barriers. It is shown that when the time taken by the wave to travel the distance between the electrodes is half the wave attenuation time in the unbounded plasma, an undamped wave may occur as a result of the superposition of waves reflected from the electrodes on the perturbation wave.

In many gas-discharge and semiconductor diodes there are regions of quasineutral plasma bounded by potential barriers which create favorable conditions for the formation of standing waves in the plasma. Studies [1–5] are devoted to the investigation of these waves in an electron plasma between plane electrodes. In all these instances, however, the conditions at the two boundaries were assumed to be the same. In fact, in a glow or arc-discharge plasma, and also in semiconductor diodes when current passes through the diode, the conditions at the boundaries between the plasma and the space-charge regions are not the same. For example, in a p-i-n diode in the forward-current, mode holes and electrons from the i region that reach the boundary between the i region and the n and p regions behave differently. Holes are reflected from the boundary between the i and the p regions and readily pass into the n region, where they recombine; electrons, on the other hand, readily pass into the p region, where they recombine, but are reflected from the boundary between the i and the n regions. A similar asymmetry of the boundary conditions occurs at the boundaries between the positive column and the electrode barriers in glow and arc discharges, as well as in a cesium diode plasma. This paper examines the longitudinal high-frequency plasma oscillations in junction diodes by the kinetic equation method with allowance for the asymmetry of the boundary conditions.

We will consider a plasma located between plane electrodes. We denote the distance between the boundaries of the plasma with the space-charge regions near the electrodes by $2l$ and select a coordinate origin at the center of the plasma, directing the x axis from the anode (collector) to the cathode (emitter). In the linear approximation the one-dimensional problem of longitudinal plasma oscillations reduces to the integration of the system of equations

$$\frac{\partial f_\alpha}{\partial t} + u\frac{\partial f_\alpha}{\partial x} + \frac{e_\alpha}{m_\alpha} E \frac{\partial f_{\alpha 0}}{\partial u} = -\frac{f_\alpha}{\tau_\alpha}, \tag{1}$$

$$\frac{\partial E}{\partial t} = \frac{4\pi e}{\varepsilon} \int\limits_{-\infty}^{\infty} (f_2 - f_1)\, du. \tag{2}$$

Here, $f_{\alpha 0}$ is the unperturbed distribution function for electrons ($\alpha = 1$) and ions or holes ($\alpha = 2$), f_α are small perturbations ($f_\alpha \ll f_{\alpha 0}$), e_α, m_α are the charge and effective mass of the current carriers, τ_α their momentum relaxation time, and E the perturbation of the electric field. The latter cannot be assumed small as compared with the unperturbed external field E_0; therefore in Eqs. (1) we have neglected the term $\sim E_0 \partial f_\alpha/\partial u$, since it is of the same order as the term $\sim E\partial f_\alpha/\partial u \ll E df_{\alpha 0}/\partial u$. Here, the effect of ionization and recombination on the high-frequency plasma oscillations is not taken into account, since the external field E_0 in the plasma is too weak for impact ionization, and the lifetime of the current carriers is several orders greater than the period of the high-frequency oscillations. The plasma of a glow or arc discharge and the plasma in the i region of a p-i-n diode in the forward mode are strongly ionized [6, 7]. This means that the electron density in such a plasma is so high that interelectronic collisions become important in energy transfer between particles, and an electron temperature is established. However, the drift velocity of the electrons in the field becomes much less than their thermal velocity. In such a plasma the unperturbed carrier distribution function can be described in the form of a Maxwell function displaced in velocity space by the drift velocity w_α:

$$f_{\alpha 0} = n_\alpha \left(\frac{m_\alpha}{2\pi\varkappa T_\alpha}\right)^{1/2} \exp\left(-\frac{m_\alpha (u - w_\alpha)^2}{2\varkappa T_\alpha}\right). \tag{3}$$

In a collisionless plasma, for example, in the drift mode of a cesium diode, the unperturbed distribution function can also be taken in the form (3), and in this case

$$w_\alpha = \left(\frac{\varkappa T_\alpha}{2\pi m_\alpha}\right)^{1/2}.$$

We will integrate Eqs. (1) and (2) for the following initial and boundary conditions:

$$f_2\,|_{t=0} = q_2(u^2,\ x), \tag{4}$$

$$f_1(-u) = f_1(u), \qquad f_2(u)|_{u<0} = 0 \qquad \text{at } x = l, \tag{5}$$

$$f_1(u)|_{u>0} = 0, \qquad f_2(-u) = f_2(u) \qquad \text{at } x = -l. \tag{6}$$

Condition (4) means that the perturbation is chosen to depend on the magnitude of the velocity, but not on its direction. Condition (5) means that electrons entrained by the perturbation wave and reaching the cathode boundary of the plasma are reflected from that boundary, while ions (holes) pass through the boundary and recombine at the cathode without returning to the plasma. Condition (6) means that ions (holes) entrained by the perturbation wave and reaching the anode boundary of the plasma are reflected from that boundary, while electrons pass through the boundary and disappear at the anode. The reflection of electrons and ions (holes) is assumed to be instantaneous and from a plane, not from a layer. This assumption is justified if the reflection time is much less than the period of the plasma oscillations, and the width of the reflecting layer is much less than the wavelength. It will be satisfied the more closely, the higher the potential barrier and the narrower the space-charge region. Taking the finiteness of the layer into account leads to additional attenuation of the order of or less than the attenuation due to collisions with neutrals.

We will not place any restrictions on the perturbed field. We merely assume that the field E(x, t) can be expanded in a Fourier series:

$$E(x,t)=\sum_{k=-\infty}^{\infty} E(k,t)\,e^{ikx},$$

$$E(k,t)=\frac{1}{2l}\int_{-l}^{l} E(x,t)\,e^{-ikx}\,dx$$

$$(lk=n\pi,\ n \text{— is an integer}). \tag{7}$$

For initial and boundary conditions (4)–(7) the system of equations (1), (2) is solvable.

We integrate Eqs. (1), (2) by means of Laplace transformations with respect to time and Fourier transformations with respect to the coordinates.

Using the Laplace transformation, instead of (1), (2) we obtain

$$qf_{q\alpha}+u\frac{\partial f_{q\alpha}}{\partial x}+\frac{e_\alpha}{m_\alpha}E_q\frac{\partial f_{\alpha 0}}{\partial u}+\frac{f_{q\alpha}}{\tau_\alpha}=g_\alpha, \tag{8}$$

$$\frac{\partial E_q}{\partial x}=\frac{4\pi e}{\varepsilon}\int_{-\infty}^{\infty}(f_{q2}-f_{q1})\,du \tag{9}$$

$$\left(f_{q\alpha}=\int_0^\infty e^{-qt}f_\alpha\,dt,\quad E_q=\int_0^\infty e^{-qt}E\,dt,\quad \operatorname{Re} q>0\right).$$

Integrating Eq. (8) with respect to the coordinate with boundary conditions (5), (6), we obtain

$$f_{q1}=-\exp\frac{-q_1(x-l)}{u}\int_{-l}^{x}\left[\frac{e_1E_q}{m_1}\frac{\partial f_{10}(u-w_1)}{u\,\partial u}-g_1\right]\exp\frac{q_1(y-l)}{u}\,dy\quad (u>0)$$

$$f_{q1}=\exp\frac{-q_1(x-l)}{u}\left\{\int_{x}^{l}\left[\frac{e_1E_q}{m_1}\frac{\partial f_{10}(u-w_1)}{u\,\partial u}-g_1\right]\exp\frac{q_1(y-l)}{u}\,dy-\int_{-l}^{l}\left[\frac{e_1E_q}{m_1}\times\frac{\partial f_{10}(u+w_1)}{u\,\partial u}-g_1\right]\exp\frac{-q_1(y-l)}{u}\,dy\right\}\quad (u<0), \tag{10}$$

$$f_{q2}=\exp\frac{-q_2(x+l)}{u}\int_{x}^{l}\left[\frac{e_2E_q}{m_2}\frac{\partial f_{20}(u-w_2)}{u\,\partial u}-g_2\right]\exp\frac{q_2(y+l)}{u}\,dy\quad (u<0).$$

$$f_{q2}=-\exp\frac{-q_2(x+l)}{u}\left\{\int_{-l}^{x}\left[\frac{e_2E_q}{m_2}\frac{\partial f_{20}(u-w_2)}{u\,\partial u}-g_2\right]\exp\frac{q_2(y+l)}{u}\,dy-\int_{-l}^{l}\left[\frac{e_2E_q}{m_2}\times\frac{\partial f_{20}(u+w_2)}{u\,\partial u}-g_2\right]\exp\frac{-q_2(y+l)}{u}\,dy\right\}\quad (u>0)$$

$$(q_\alpha=q+1/\tau_\alpha). \tag{11}$$

Using (10), (11), we write Eq. (9) thus:

$$\frac{\partial E_q(x)}{\partial x}=\frac{4\pi e^2}{\varepsilon}\left\{\int_x^l[K_1^+(y-x)+K_2^+(y-x)]\,E_q(y)\,dy-\int_{-l}^{x}[K_1^-(x-y)+K_2^-(x-y)]\,E_q(y)\,dy+\int_{-l}^{l}[K_2^+(2l+x+y)-K_1^-(2l-x-y)]\times E_q(y)\,dy\right\}+\Phi(g_1,g_2),$$

$$K_\alpha^\pm(\zeta)=\frac{1}{m_\alpha}\int_0^\infty\frac{1}{u}\exp\frac{-q_\alpha\zeta}{u}\frac{\partial f_{\alpha 0}(u\pm w_\alpha)}{\partial u}\,du. \tag{12}$$

Here, $\Phi(g_1, g_2)$ is the initial perturbation function. We formally continue $K_\alpha^\pm(\zeta)$ into the region $\zeta<0$ with the aid of the relation

$$K_\alpha^\pm(-\zeta)=-K_\alpha^\mp(\zeta). \tag{13}$$

Using (13), we write Eq. (12) in the more compact form

$$\frac{\partial E_q(x)}{\partial x}=\frac{4\pi e^2}{\varepsilon}\int_{-l}^{l}\{K_2^+(2l+x+y)-K_1^-(2l-x-y)-K_2^-(x-y)-K_1^-(x-y)\}\,E_q(y)\,dy+\Phi(g_1,g_2), \tag{14}$$

and solve it by the Fourier method. Then Eq. (14) is written as a system of algebraic equations

$$E_q(k)\,[1+A_1(k)+A_2(k)]+\sum_{k_1}E_q(k_1)\,[B_1(-k,-k_1)+B_2(k,k_1)]=\Phi_k, \tag{15}$$

where

$$E_q(k)=\frac{1}{2l}\int_{-l}^{l}E_q(x)\,e^{-ikx}\,dx$$

and k_1 satisfies the same condition (7) as k,

$$A_\alpha(k)=\frac{4\pi e^2}{ikm_\alpha}\int_{-\infty}^{\infty}\frac{\partial f_{\alpha 0}(u-w_\alpha)}{\partial u}\frac{1}{q_\alpha+iku}\,du, \tag{16}$$

$$B_\alpha(k,k_1)=\frac{4\pi e^2}{iklm_\alpha}\int_0^\infty\left\{\frac{\operatorname{sh}[(q_\alpha-ik_1u)\,l/u]}{q_\alpha-ik_1u}\times\frac{\partial f_\alpha(u+(-1)^\alpha w_\alpha)}{\partial u}\left[\frac{\exp(iku-q_\alpha)\,l/u}{q_\alpha-iku}-2\,\frac{\exp(-2q_\alpha l/u)\operatorname{sh}[(q_\alpha+iku)\,l/u]}{q_\alpha+iku}\right]-\frac{\operatorname{sh}[(q_\alpha+ik_1u)\,l/u]\,\exp[-(iku+q_\alpha)\,l/u]}{(q_\alpha+ik_1u)(q_\alpha+iku)}\times\frac{\partial f_{\alpha 0}(u-(-1)^\alpha w_\alpha)}{\partial u}\right\}u\,du\quad(\operatorname{Re}q_\alpha>0). \tag{17}$$

The dependence of the field on time can be found for each Fourier component using the inversion formula

$$E(k,t)=\frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma}e^{qt}E_q(k)\,dq\qquad(\sigma>0),$$

where $E_q(k)$ is the solution of Eq. (15) determined in the right half-plane of q. Analytically continuing $E_q(k)$ into the left half-plane of q, we see that the only singularities of $E_q(k)$ are the poles representing the roots of the equation

$$\operatorname{Det}|[1 + A_1(k) + A_2(k)]\delta_{kk_1} + B_1(-k, -k_1) + B_2(k, k_1)| = 0. \tag{18}$$

At large t the solution E(k, t) will be proportional to e^{qt}, where $q = -i\omega - \gamma$ is the root of Eq. (16) having the greatest real part. Here, ω is the oscillation frequency, and γ the attenuation constant. Equation (16) is the dispersion equation of the oscillations of a bounded plasma in an external electric field; it is valid for both high-frequency and low-frequency plasma oscillations. As $l \to \infty$ the terms B_α in Eq. (18) vanish and the latter takes the form

$$1 + A_1(k) + A_2(k) = 0. \tag{19}$$

Equation (19) coincides with Eq. (10) of [8] derived for the oscillations of an unbounded plasma in an external electric field.

We will consider high-frequency plasma oscillations whose frequency is of the same order as the Langmuir frequency of the electron oscillations

$$\omega \approx \omega_1 = \left(\frac{4\pi e^2 n_1}{\varepsilon m_1}\right)^{1/2}.$$

We assume that the phase velocity ω/k of the waves is much greater than the thermal velocity of the particles $s_\alpha = (\varkappa T_\alpha/m_\alpha)^{1/2}$. It does not make sense to consider the problem of high-frequency plasma oscillations with boundary conditions at a wave phase velocity comparable with or less than the electron thermal velocity, since in this case the Landau damping is comparable with or greater than the frequency of the oscillations, and the boundary conditions have no effect on them.

We introduce the dimensionless variable and parameters

$$U = \frac{u}{s_\alpha}, \qquad v_\alpha = \frac{w_\alpha}{s_\alpha}, \qquad \beta_\alpha = \frac{iq_\alpha}{|k|\, s_\alpha}.$$

In these variables for the waves considered we can represent the quantity $A_\alpha(k)$ in the form of an asymptotic series [8]

$$A_\alpha(k) = -\frac{1}{k^2 a_\alpha^2}\left(\frac{1}{\gamma_\alpha^2} + \frac{3}{\gamma_\alpha^4} + \ldots\right) + i\left(\frac{\pi}{2}\right)^{1/2} \frac{\gamma_\alpha}{k^2 a_\alpha^2} \exp\frac{-\gamma_\alpha^2}{2}$$

$$\left(a_\alpha = \left(\frac{\varepsilon \varkappa T_\alpha}{4\pi e^2 n_\alpha}\right)^{1/2}, \qquad \gamma_\alpha = \frac{|k|}{k}\beta_\alpha - v_\alpha\right) \tag{20}$$

where a_α is the Debye radius of the particle α. In the integral B_α we will confine ourselves to linear terms of the expansion of the integrand in powers of $v_\alpha \ll 1$ and represent it in the form of a sum of three integrals:

$$B_\alpha(k, k_1) = (-1)^{n-m} \frac{i}{k^2 a_\alpha^2 kl}(I_0 + I_1 + I_2),$$

$$I_0 = \frac{1}{\sqrt{2\pi}} \int_0^\infty \left[C_\alpha(u) - \frac{1}{2} D_\alpha(u)\right] e^{-1/2 u^2}\, du,$$

$$I_1 = \frac{1}{\sqrt{2\pi}} \int_0^\infty [-C_\alpha(u) + D_\alpha(u)]\, e^{\varphi_1(u)}\, du,$$

$$I_2 = -\frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{e^{\varphi_\alpha(u)}}{(\beta_\alpha + uk_1/|k|)(\beta_\alpha - uk/|k|)}\, du,$$

$$C_\alpha = \Big[(-1)^\alpha (\beta_\alpha^2 + u^2 k_1/|k|)\, u (1 - u^2) v_\alpha - u^3 \beta_\alpha (k + k_1)/|k|\Big]\Big[(\beta_\alpha^2 - u^2 k_1^2/|k^2|)(\beta_\alpha^2 - u^2)\Big]^{-1},$$

$$D_\alpha = \frac{u^2 + (-1)^\alpha v_\alpha (1 - u^2)\, u}{(\beta_\alpha + uk_1/|k|)(\beta_\alpha - uk/|k|)},$$

$$\varphi_1(u) = \frac{2i|k|\, l\beta_\alpha}{u} - \frac{u^2}{2},$$

$$\varphi_2(u) = \frac{4i|k|\, l\beta_\alpha}{u} - \frac{u^2}{2},$$

$$n = kl/\pi, \qquad m = k_1 l/\pi,$$

$$\operatorname{Re}\beta_\alpha > 0, \quad \operatorname{Im}\beta_\alpha > 0, \quad \operatorname{Re}\beta_\alpha \gg \operatorname{Im}\beta_\alpha. \tag{21}$$

The integral I_0 is easily evaluated. Asymptotically at $\beta_\alpha \gg 1$ it is equal to

$$I_0 = \frac{1}{\beta_\alpha^2}\left[\frac{1}{4} + (-1)^\alpha \frac{v_\alpha}{2\sqrt{2\pi}}\right]. \tag{22}$$

In order to evaluate the second integral we employ the stationary phase method. For this purpose we replace $\varphi_1(u)$ with the first terms of the series expansion in powers of $(u - u_1)$:

$$\varphi_1(u) = \varphi_1(u_1) + {}^1/_2\, \varphi_1''(u_1)(u - u_1)^2,$$

where u_1 is the root of the equation $\varphi'(u_1) = 0$:

$$u_1 = e^{-1/6 i\pi} \sqrt[3]{2|k|\, l\, |\beta_\alpha|} \tag{23}$$

and

$$\varphi_1(u_1) = -{}^3/_2\, u_1^2, \qquad \varphi_1''(u_1) = -3. \tag{24}$$

We take the integration path parallel to the real axis passing through the point $u = u_1$. The singular points $u = \beta_\alpha$ and $u = \beta_\alpha k_1/k$ will lie above this path even at $\operatorname{Im}\beta_\alpha < 0$, if $|\operatorname{Im}\beta_\alpha| < |\operatorname{Im} u_1|$. We assume that the latter inequality is satisfied, since we are considering oscillations for which $u = \beta_\alpha$ is close to the real axis. Evaluating the integral I_1 by the stationary phase method, we obtain

$$I_1 = \frac{D_\alpha(u_1) - C_\alpha(u_1)}{\sqrt{3}} e^{-3/2 u_1^2}. \tag{25}$$

Obviously, at $|\beta_\alpha| \gg 1$ the modulus of the square $|u_1^2|$ is much greater than unity at all values of $n = kl/\pi$ and is the greater, the greater n. Therefore the correction I_1 will be significant only for integers n close to unity. For these n, obviously, $|\beta_\alpha^2| \gg |u_1^2|$, and from (25) we have approximately

$$I_1 = \frac{|u_1^2|}{2\sqrt{3}\,|\beta_\alpha^2|}\, e^{-3/4 u_1^2}\left\{\cos\left(\frac{3\sqrt{3}}{4}|u_1^2|\right) + \right.$$
$$+ 3\sin\left(\frac{3\sqrt{3}}{4}|u_1^2|\right) + i\left[\sin\left(\frac{3\sqrt{3}}{4}|u_1^2|\right) - \right.$$
$$\left.\left. - \sqrt{3}\cos\left(\frac{3\sqrt{3}}{4}|u_1^2|\right)\right]\right\}. \tag{26}$$

A similar formula is obtained for I_2, but in this case it is necessary to replace

$$|u_1^2| \text{ by } |u_2^2| = \sqrt[3]{2}\,|u_1^2|.$$

We note that the real parts of I_1 and I_2 are much smaller than the real part of I_0; therefore they can be neglected in expression (21) and only the imaginary parts retained. Thus, we can finally write

$$B_\alpha(k, k_1) \approx \frac{(-1)^{n-m}}{k^2 a_\alpha^2 \beta_\alpha^2 kl}\left\{\sum_{\delta=1}^{2} \frac{|u^2_\delta|}{2\sqrt{3}} \times \right.$$
$$\times \left[\sqrt{3}\cos\left(\frac{3\sqrt{3}}{4}|u_\delta^2|\right) - \sin\left(\frac{3\sqrt{3}}{4}|u_\delta^2|\right)\right] \times$$
$$\left. \times e^{-3/4|u_\delta^2|} - i\left[\frac{1}{4} + (-1)^\alpha \frac{v_\alpha}{2\sqrt{2\pi}}\right]\right\}. \tag{27}$$

Substituting (20) and (27) into Eq. (16) makes it possible to solve the problem of determining the frequency and logarithmic decrement in a junction diode.

We will consider the electronic plasma oscillations in a gas-discharge junction diode. In determining the frequency and logarithmic decrement of the electronic plasma oscillations it is possible to disregard the motion of the ions and set $A_2 = B_2 = 0$ in Eq. (16). In the first approximation we will not take into account the real part of the correction $B_1(-k, -k_1)$. Going over to dimensional quantities, we introduce the notation

$$A_1'(k) \equiv -\operatorname{Re} A_1(k) = \frac{\omega_i^2}{(\omega - wk)^2}(1 + 3k^2 a_1^2),$$
$$A_1''(k) \equiv \operatorname{Im} A_1 = \frac{\gamma_0 + 1/\tau_1 - \gamma}{\omega_i},$$
$$B_1''(k) \equiv (-1)^{n-m} \times$$
$$\times \operatorname{Im} B_1(-k, -k_1) = \frac{1}{kl}\left(\frac{1}{4} + \frac{|w_1|}{2\sqrt{2\pi s_1}}\right).$$

We write Eq. (16) in the form

$$\Delta = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & a_{k-1,\,l-1} & a_{k-1,\,l} & a_{k-1,\,l+1} & \cdots \\ \cdots & a_{k,\,l-1} & a_{k,\,l} & a_{k,\,l+1} & \cdots \\ \cdots & a_{k+1,\,l-1} & a_{k+1,\,l} & a_{k+1,\,l+1} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix} = 0$$

$$a_{k-1,\,l-1} = 1 - A_1'(k-1) +$$
$$+ i\,[A_1''(k-1) + \beta_1''(k-1)],$$
$$a_{k-1,l} = -i\beta_1''(k-1),$$
$$a_{k-1,\,l+1} = i\beta_1''(k-1),$$
$$a_{k,\,l-1} = -i\beta_1''(k),$$
$$a_{k,\,l} = 1 - A_1'(k) + i\,[A_1''(k) + \beta_1''(k)]$$
$$a_{k,\,l+1} = -i\beta_1''(k)$$
$$a_{k+1,\,l-1} = i\beta_1''(k+1),$$
$$a_{k+1,\,l} = -i\beta_1''(k+1)$$
$$a_{k+1,\,l+1} = 1 - A_1'(k+1) +$$
$$+ i\,[A_1''(k+1) + \beta_1''(k+1)]. \tag{28}$$

Isolating the real part of the determinant and considering that $A_1''(k) \ll A_1'(k)$, we obtain

$$\operatorname{Re}\Delta = \prod_k (1 - A_1'(k)) = 0. \tag{29}$$

As may be seen from (29), the corrections B_1'' do not enter into the real part of the determinant. This means that in the approximation considered the presence of boundaries does not affect the oscillation frequency and for a given wave number k the frequency is found from the equation

$$1 - A_1'(k) = 0 \tag{30}$$

whose solution is given by the Vlasov formula

$$\omega = \omega_1(1 + {}^3/_2\, k^2 a_1^2) + (\mathrm{wk}). \tag{31}$$

In the next approximation a small correction equal to $-1/2$, $\operatorname{Re} B_1(-k, -k_1)$ is added to the frequency. In determining the imaginary part of the determinant we note that the product of the off-diagonal terms iB_1'' is compensated by the product of the same terms in the diagonal elements of the determinant. Consequently,

$$\operatorname{Im}\Delta = \sum_k [A_1''(k) + B_1''(k)] \times$$
$$\times \prod_{k' \neq k} (1 - A_1'(k') + O(B_1''^3)). \tag{32}$$

For those k for which Eq. (30) is satisfied, and consequently

$$\prod_{k' \neq k} (1 - A_1'(k')) \neq 0$$

and from (32) we obtain

$$A_1''(k) + B_1''(k) = 0. \tag{33}$$

From this we find the wave attenuation

$$\gamma = \gamma_0 + \frac{1}{\tau_1} + \frac{\omega_1}{2kl}\left(\frac{1}{2} + \frac{|w_1|}{\sqrt{2\pi s_1}}\right). \tag{34}$$

As may be seen from (34), the presence of plasma boundaries affects the wave attenuation in such a way that a wave propagating from the anode to the cathode (k > 0) is more strongly, and one propagating from cathode to the anode (k < 0) less strongly damped than a wave in the unbounded plasma. The lesser attenuation of a wave moving from the cathode to the anode is associated with the fact that on it there is superimposed a wave due to the same perturbation reflected from the cathode and having the same wavelength and phase velocity. In this case when the time taken by the wave to travel the distance between the electrodes is half the wave attenuation time in the unbounded plasma

$$t \equiv \frac{2kl}{\omega} \approx \frac{1}{2} t_1 \equiv \frac{1}{2} \frac{\tau_1}{1 + \gamma_0 \tau_1} \tag{35}$$

a wave moving from the cathode to the anode becomes continuous in the course of the time interval during which the perturbation acts. The electron drift velocity plays only an unimportant part, since $|w_1|/s_1 \ll 1$. However, it still helps to build up an undamped wave moving in the direction of electron drift.

In a semiconductor diode it is necessary to take hole oscillations into account as well as electron oscillations. In a semiconductor diode for the high-frequency oscillations we have

$$\omega = \sqrt{\omega_1^2 + \omega_2^2} + \frac{3}{2} \frac{k^2 (\omega_1^2 s_1^2 + \omega_2^2 s_2^2)}{\omega_1^2 + \omega_2^2} + (\mathbf{w}_1 \mathbf{k}) + (\mathbf{w}_2 \mathbf{k}), \tag{36}$$

$$\gamma = \frac{\omega_1^2 / \tau_1 + \omega_2^2 / \tau_2}{\omega_1^2 + \omega_2^2} + \left(\frac{\pi}{2}\right)^{1/2} (\omega_1^2 + \omega_2^2) \left(\exp\left[-\frac{\omega_1^2 + \omega_2^2}{\omega_1^2 k^2 a_1^2} \right] \times \frac{1}{\omega_1 k^3 a_1^3} + \exp\left[-\frac{\omega_1^2 + \omega_2^2}{\omega_2^2 k^2 a_2^2} \right] \frac{1}{\omega_2 k^3 a_2^3} \right) + \frac{\omega_1^2 (1/2 + |w_1| / \sqrt{2\pi} s_1) - \omega_2^2 (1/2 + |w_2| / \sqrt{2\pi} s_2)}{2kl \sqrt{\omega_1^2 + \omega_2^2}}. \tag{37}$$

As may be seen from (36), boundaries have no effect on the oscillation frequency. As may be seen from (37), boundaries likewise have no effect on the wave attenuation if $\omega_1 = \omega_2$, which holds when the effective masses of electrons and holes are equal $m_1 = m_2$ in an intrinsic semiconductor (for example, in the i region of a p-i-n diode) or when $n_1/m_1 = n_2/m_2$ in a doped semiconductor. However, if $\omega_1 \gg \omega_2$, we have purely electronic plasma oscillations, possibly with satisfaction of condition (35), when undamped oscillations propagating from the emitter to the collector are observed. Since $1/\tau_1 \approx 10^{12}$ sec^{-1}, while for satisfaction of condition (35) it is necessary for ω_1 to be greater than $1/\tau_1$ by at least an order, the electron density must be $n_1 > 4 \cdot 10^{17} m_1/m_0$ cm^{-3}, where m_0 is the mass of the free electron. However, if $\omega_2 \gg \omega_1$, we have purely hole oscillations, for which there may also be satisfaction of condition (35) with undamped oscillations propagating from the collector to the emitter. Since $1/\tau_2 \approx 10^{12}$ sec^{-1}, for condition (35) to be satisfied it is necessary that $n_2 > 4 \cdot 10^{17} m_2/m_0$ cm^{-3}.

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